International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 4, Issue 3, Apr - May 2015, 1-8

International Academy of Science, Engineering and Technology
IASET Connecting Researchers; Nurturing Innovations

A STOCHASTIC MODEL FOR ESTIMATION OF VARIANCE OF TIME TO RECRUITMENT FOR A SINGLE GRADE MANPOWER SYSTEM WITH DIFFERENT EPOCHS FOR DECISIONS AND EXITS

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ABSTRACT

In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment for a single grade manpower system in which attrition takes places due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times form a geometric process and inter-exit times form an ordinary renewal process. The analytical results are numerical illustrated and the effect of the nodal parameters on the performance measures is studied.

KEYWORDS: Single Grade Manpower System, Decision and Exit Epochs, Geometric Process, Ordinary Renewal Process, Univariate Policy of Recruitment and Variance of Time to Recruitment

AMS Mathematics Subject Classification (2010): 60K05, 60K10, 62N05

1. INTRODUCTION

Attrition is a common phenomenon in many marketing organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the breakdown threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. Many researchers have studied several problems in manpower planning using different methods. In [1] and [2] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [17] the authors have analyzed the problem of time to recruitment for a single grade manpower system which is subjected to attrition, using the univariate policy of recruitment based on shock model approach for replacement of systems in reliability theory. In [14] the author has studied the problem of time to recruitment for a single grade manpower system and obtained the variance of the time to recruitment when the loss of manpower forms a sequence of independent and identically distributed random variables, the inter- decision times form a geometric process and the mandatory breakdown threshold for the cumulative loss of manpower is an exponential random variable by using the univariate cum policy of recruitment. In [19] the author has studied the work in [14] using univariate and bivariate policies of recruitment both for the exponential mandatory threshold and for the one whose distribution has the SCBZ property. In [12] the authors have analyzed the work in [14] with (i) exponential breakdown threshold and (ii) extended exponential threshold having shape parameter 2 using the bivariate cum policy of recruitment. In [10] the author has studied the work in [19] when the recruitment policies involve optional and mandatory thresholds. In [13] the authors have studied the problem of time to recruitment for a single grade manpower system by assuming that the attrition is generated by a geometric process of inter- decision times using a different probabilistic analysis.

In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. This aspect is taken into account for the first time in [3] and the variance of the time to recruitment is obtained when the loss of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables according as the mandatory breakdown threshold is an exponential random variable or extended exponential random variable with shape parameter 2 or a continuous random variable with SCBZ property. In [5] the authors have studied the work in [3] when the inter-decision times form a sequence of exchangeable and constantly correlated exponential random variables. In [4], [6], [7] and [8] the authors have extended the research work in [3] when the inter-decision times form (i) an ordinary renewal process (ii) a sequence of exchangeable and constantly correlated exponential random variables (iii) a geometric process and (iv) an order statistics respectively using a different probabilistic analysis. Recently in [9] the authors have studied the results of [4], [6], [7], and [8] using univariate Max policy of recruitment. The present paper extends the research work in [5] when the inter-policy decision times form a geometric process.

2. MODEL DESCRIPTION

Consider an organization taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) due to i^{th} exit point with probability distribution function M(.), density function m(.) and mean $\frac{1}{\alpha}(\alpha>0)$. Let S_k be the cumulative loss of manpower in the first k exit points. It is assumed that A_i , the time between $(i-1)^{th}$ and i^{th} exits, form a geometric process of independent random variables and c (c>0) is the parameter of this geometric process. Let F(.) and f(.) be the distribution and probability density function of A_1 respectively. Let B_i , the time between the $(i-1)^{th}$ and i^{th} exit times, be independent and identically distributed random variables with probability distribution function G(.), density function g(.) and mean $\frac{1}{\delta}$ $(\delta>0)$. Let $V_k(t)$ be the probability that there are exactly k exit points in (0,t]. Let Y be the independent breakdown threshold level for the cumulative depletion of manpower in the organization with probability distribution function H(.) and density function h(.). Let q be the probability that every policy decision has exit of personnel. Since q=0 means exits due to policy decisions are impossible, it is assumed that $q\neq 0$. Let T be the random variable denoting the time to recruitment with probability distribution function L(.), density function l(.) , mean E(T) and variance V(T). Let $\tilde{U}(.)$ and $u^*(.)$ be the Laplace - Stieltjes transform and the Laplace transform of U (.) and u (.) respectively. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold.

3. MAIN RESULT

Our recruitment policy together with law of total probability give the following expression for the tail distribution of T.

$$P(T > t) = \sum_{K=0}^{\infty} V_K(t)\, P(S_k < Y) \tag{1}$$

We now obtain the distribution and the variance of time to recruitment by considering different threshold distributions.

Case (i): H(x) = 1- $e^{-\theta x}$

In this case from (1) we get

$$P(T > t) = \sum_{k=0}^{\infty} \left[G_k(t) - G_{k+1}(t)\right] a^k \tag{2}$$

$$L(t) = \overline{a} \sum_{k=1}^{\infty} G_k(t)\, a^{k-1} \tag{3}$$

and

$$\tilde{L}(s) = \frac{\overline{a}\, \tilde{G}(s)}{1 - a\, \tilde{G}(s)} \tag{4}$$

where a = E[$e^{-\theta X}$] and $\overline{a}$ = 1- a (5)

It can be shown that the distribution function G(.) of the inter-exit times satisfy the relation

$$G(x) = q \sum_{n=1}^{\infty} (1-q)^{n-1} F_n(x) \tag{6}$$

Therefore from (4) and (6) we get on simplification that

$$E(T) = \frac{c}{\overline{a}\lambda(c-1+q)} \tag{7}$$

and

$$V(T) = \frac{2\overline{a}c^3(c-1+q) + (c^2-1+q)(ac^2-1)}{(\overline{a})^2\lambda^2(c^2-1+q)(c-1+q)^2} \tag{8}$$

where $\overline{a}$ is giving by (5)

(7) and (8) give the mean and variance of the time to recruitment for case(i).

Case (ii): $H(x) = [1-e^{-\theta x}]^2$ which is the extended exponential distribution with scale parameter θ and shape parameter two [11].

In this case it can be shown that

$$\mathrm{L(t)} = 2\bar{a} \sum_{k=1}^{\infty} G_k(\mathrm{t})\,\mathrm{a}^{k-1} - \bar{b}\sum_{k=1}^{\infty} G_k(\mathrm{t})\,\mathrm{b}^{k-1} \tag{9}$$

and

$$\tilde{L}(\mathrm{s}) = \frac{2\bar{a}\,\tilde{G}(\mathrm{s})}{1-a\,\tilde{G}(\mathrm{s})} - \frac{\bar{b}\,\tilde{G}(\mathrm{s})}{1-b\,\tilde{G}(\mathrm{s})} \tag{10}$$

where $\bar{a}$ is given by (5) and $\bar{b}$ =1- E[$e^{-2\theta X}$] (11)

Therefore from (6) and (10) we get on simplification that

$$E(T) = \frac{c}{\lambda(c-1+q)}\left[\frac{2\bar{b}-\bar{a}}{\bar{a}\bar{b}}\right] \tag{12}$$

and

$$V(T) = \left(\frac{1}{c^2-1+q}\right)\left(\frac{c}{\bar{a}\bar{b}\,\lambda(c-1+q)}\right)^2 \begin{Bmatrix} 2\left[(c-1+q)(2c\bar{a}(\bar{b})^2 - c(\bar{a})^2\bar{b}) + (c^2-1+q)(2a(\bar{b})^2 - (\bar{a})^2 b)\right] - \\ (c^2-1+q)(2\bar{b}-\bar{a})^2 \end{Bmatrix} \tag{13}$$

(12) and (13) give the mean and variance of the time to recruitment for case(ii).

Case (iii): H(x) = $p_1 e^{-(\theta_1+\mu)\mathrm{x}} + p_2 e^{-\theta_2 x}$, which is the distribution function with SCBZ property [16].

$$\text{where } p_1 = \frac{\theta_1-\theta_2}{\mu+\theta_1-\theta_2} \text{ and } p_2 = 1-p_1 \tag{14}$$

In this case it can be shown that

$$\mathrm{L(t)} = p_1\bar{a}_2 \sum_{k=1}^{\infty} G_k(\mathrm{t})\,\mathrm{a}_2^{\,k-1} + p_2\bar{b}_2 \sum_{k=1}^{\infty} G_k(\mathrm{t})\,\mathrm{b}_2^{\,k-1} \tag{15}$$

$$\tilde{L}(\mathrm{s}) = \frac{p_1\bar{a}_2\tilde{G}(\mathrm{s})}{1-a_2\tilde{G}(\mathrm{s})} + \frac{p_2\bar{b}_2\tilde{G}(\mathrm{s})}{1-b_2\tilde{G}(\mathrm{s})} \tag{16}$$

where $\bar{a}_2$ = 1- E$\left[e^{-(\theta_1+\mu)\mathrm{X}}\right]$, $\bar{b}_2$ = 1- E$\left[e^{-\theta_2\mathrm{X}}\right]$ (17)

Therefore from (6) and (16) we get on simplification that

$$E(T) = \left(\frac{c}{\lambda(c-1+q)}\right)\left[\frac{p_1\bar{b}_2 + p_2\bar{a}_2}{\bar{a}_2\bar{b}_2}\right] \tag{18}$$

and

$$\mathrm{V}(T)=\left(\frac{c}{\bar{a}_2\bar{b}_2\lambda(c-1+q)}\right)^2\left[\left(\frac{2}{c^2-1+q}\right)c\bar{a}_2\bar{b}_2(c-1+q)(p_1\bar{b}_2+p_2\bar{a}_2)+2(p_1a_2(\bar{b}_2)^2+p_2(\bar{a}_2)^2b_2)\right]-\left[(p_1)^2(\bar{b}_2)^2+(p_2)^2(\bar{a}_2)^2+2p_1p_2\bar{a}_2\bar{b}_2\right] \tag{19}$$

(18) and (19) give the mean and variance of the time to recruitment for case(iii).

3.1 Note

i. When q=1, our results agree with the results in [14] for all the three cases.

ii. When c=1, our results agree with the results in [3] for all the three cases.

iii. When q=1 and c=1, our results for cases (ii) and (iii) are consistent with those of [15] and [18] respectively

4. NUMERICAL ILLUSTRATION

The mean and variance of time to recruitment for all the three cases are numerically illustrated by varying one parameter a time and keeping all the other parameters fixed. The effect of the nodal parameters on the mean and variance of time to recruitment is shown in a table. In all the cases we take θ=0.05.We consider the cases c<1 and c>1.

Table 1: (Effect of the Nodal Parameters on E(T) and V(T))

α	λ	q	c	Case (i)		Case (ii)		Case (iii)	
				E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
2	0.5	0.8	0.6	123	0.148×10^{5}	183	0.196×10^{5}	31.982	0.504×10^{4}
4	0.5	0.8	0.6	243	0.584×10^{5}	363	0.752×10^{5}	61.864	0.856×10^{4}
6	0.5	0.8	0.6	363	1.308×10^{5}	543	1.668×10^{5}	91.747	1.058×10^{4}
0.5	2	0.8	0.6	8.25	62.5556	12.0	96.750	2.392	88.291
0.5	4	0.8	0.6	4.125	15.6389	6.00	24.187	1.196	22.072
0.5	6	0.8	0.6	2.750	6.9506	4.00	10.750	0.797	9.810
0.5	0.5	0.8	0.6	33.0	1.000×10^{3}	48.0	1.548×10^{3}	9.570	1.412×10^{3}
0.5	0.5	0.9	0.6	26.4	0.651×10^{3}	38.4	0.926×10^{3}	7.656	0.891×10^{3}
0.5	0.5	1	0.6	22.0	0.455×10^{3}	32.0	0.624×10^{3}	6.380	0.615×10^{3}
2	0.5	0.8	2.5	89.130	0.893×10^{4}	132.608	0.999×10^{4}	23.175	2.594×10^{3}
4	0.5	0.8	2.5	176.087	3.296×10^{4}	263.043	3.888×10^{4}	44.829	4.392×10^{3}
6	0.5	0.8	2.5	263.043	7.211×10^{4}	393.478	8.668×10^{4}	66.483	5.398×10^{3}
0.5	2	0.8	2.5	5.978	52.367	8.695	45.608	1.733	45.325
0.5	4	0.8	2.5	2.989	13.091	4.347	11.402	0.866	11.331
0.5	6	0.8	2.5	1.992	5.818	2.898	5.0676	0.577	5.036
0.5	0.5	0.8	2.5	23.913	837.880	34.782	729.741	6.935	725.213
0.5	0.5	0.9	2.5	22.916	772.719	33.333	673.695	6.646	666.736
0.5	0.5	1	2.5	22.000	715.000	32.000	624.000	6.380	615.086
0.5	0.5	0.8	0.9	28.285	767.184	41.142	1.038×10^{3}	8.203	1.018×10^{3}
0.5	0.5	0.8	0.8	29.333	810.951	42.666	1.130×10^{3}	8.507	1.097×10^{3}
0.5	0.5	0.8	0.7	30.800	880.867	44.800	1.274×10^{3}	8.932	1.215×10^{3}
0.5	0.5	0.3	1	73.333	4.237×10^{3}	106.666	6.933×10^{3}	21.268	6.834×10^{3}
0.5	0.5	0.3	1.5	41.250	1.755×10^{3}	60.000	2.092×10^{3}	11.963	2.142×10^{3}
0.5	0.5	0.3	2	33.846	1.391×10^{3}	49.230	1.412×10^{3}	9.8160	1.443×10^{3}

5. FINDINGS

From the above table the following observations are presented which agree with reality,

i. When α increases and keeping all the other parameter fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment increase for all the three cases.

ii. As λ increases, on the average, the inter-decision time decreases and consequently the mean and variance of time to recruitment decrease for all the three cases when the other parameters are fixed.

iii. As q, the probability that every policy decision has exit of personnel increases, the mean and variance of time to recruitment decrease for all the three cases when the other parameters are fixed.

iv. The mean and variance of the time to recruitment are decreasing or increasing according as $c>1$ or $c<1$, since the geometric process of inter-policy decision times is stochastically decreasing when $c>1$ and increasing when $c<1$.

6. CONCLUSIONS

The model discussed in this paper is found to be more realistic and new in the context of considering

(i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) a probability for any decision to have exit points and (iii) bringing monotonicity by associating geometric process for inter-policy decision times. Our model is more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points provides a better domain for the stochastic model, thereby enriching the scope of the present research work in the process.

ACKNOWLEDGEMENTS

The first author gratefully acknowledgement the University Grants Commission, India for providing financial support under Minor Research Project, Proposal Number 776, Link Number 5785, January 2015.

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